

# Coexistence of thermal noise and squeezing in the intensity fluctuations of small laser diodes

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The intensity fluctuations of laser light are derived from photon number rate equations. In the limit of short times, the photon statistics for small laser devices such as typical semiconductor laser diodes show thermal characteristics even above threshold. In the limit of long time averages represented by the low frequency component of the noise, the same devices exhibit squeezing. It is shown that squeezing and thermal noise can coexist in the multi-mode output field of laser diodes. This result implies that the squeezed light generated by regularly pumped semiconductor laser diodes is qualitatively different from single mode squeezed light. In particular, no entanglement between photons can be generated using this type of collective multi-mode squeezing.

## 1. Introduction

In the early days of laser physics, one of the most fundamental properties attributed to lasers was the reduction of photon number fluctuations below thermal levels at threshold<sup>1-5</sup>. More recently, the possibility to reduce the intensity noise of semiconductor laser diodes even below the shot noise limit has once again drawn attention to the photon number statistics of light emitted by laser devices<sup>6-12</sup>. However, in the case of semiconductor lasers, the microscopic laser dynamics depends sensitively on the charge carrier densities which provide the optical gain. Quantum theories of laser light often fail to include this dynamical degree of freedom. In particular, the theories which show the reduction of photon number fluctuations in the laser cavity below thermal noise usually rely on the adiabatic elimination of the carrier dynamics<sup>13,14</sup>. In the following, it is shown that, without this elimination of the carrier relaxation dynamics, small semiconductor lasers may exhibit thermal noise even far above threshold. In particular, the photon number fluctuation inside the cavity may even be thermal at quantum efficiencies greater than 50 %, allowing a suppression of the low frequency intensity fluctuations below the shot noise limit. This coexistence of thermal noise and squeezing can be derived theoretically from the same photon number rate equations as the thermal noise. It is therefore necessary to distinguish the photon number statistics at short times from the time integrated statistics of the low frequency noise component. Specifically, the low frequency noise is not associated with any well defined mode, but represents a collective property of many modes due to the limited temporal coherence of the emitted light. This implies that even an ideal single mode laser exhibits the anti-correlated intensity statistics observed in multi-mode lasers<sup>10,15,16</sup>. Even the squeezing observed in a perfect single mode laser can therefore include a large amount of thermal noise and does not usually correspond to the generation of a minimum uncertainty state.

The paper is organized as follows: In section 2, the rate equations are formulated. Within this framework we present our definition of the *laser threshold*. In the subsequent section 3, the photon number fluctuations are determined which allow the definition of a *noise threshold*. In section 4, the low frequency limit of intensity noise is derived and the *squeezing threshold* is defined. In section 5, the results are summarized and the conditions for a coexistence of squeezing and thermal noise is obtained. In section 6, the nature of collective multi-mode squeezing is discussed and limitations of quantum optical applications are pointed out.

## 2. Rate equations for the energy flow in lasers

A laser device converts the energy injected into the gain medium into laser light by means of light field amplification inside the laser cavity. Since the energy in the gain medium and the energy of the light field are both quantized, the energy flow cannot be entirely smooth and continuous. Instead, it is described by transition rates. Figure 1 illustrates the transition rates between the two energy reservoirs and the environment. Note that the rates given pertain to a single mode laser under the assumption of a linear carrier density dependence for gain and spontaneous emission. The laser device is thus characterized by the spontaneous emission factor  $\beta$ , the excitation lifetime  $\tau$ , the excitation number at transparency  $N_T$  and the cavity loss rate  $\kappa$ . The rate equations for the excitation number  $N$  and the cavity photon number  $n$  at an injection rate  $j$  may be formulated according to figure 1 as

$$\begin{aligned}\frac{d}{dt}N &= j - \frac{1}{\tau}N - 2\frac{\beta}{\tau}(N - N_T)n + q_N(t) \\ \frac{d}{dt}n &= 2\left(\frac{\beta}{\tau}(N - N_T) - \kappa\right)n + \frac{\beta}{\tau}N + q_n(t),\end{aligned}\tag{1}$$

where  $q_N(t)$  and  $q_n(t)$  represent the shot noise terms corresponding to the respective transitions into and out of the gain medium excitation  $N$  and the cavity photon number  $n$ , respectively. Note that equation (1) corresponds to the rate equation of Lax and Louisell<sup>3</sup>. Although a precise quantum mechanical treatment would require the solution of a master equation for both the light field and the gain medium, it is possible to reduce this equation to the diagonal elements in photon and excitation number. The statistical equations then correspond to those of a classical particle flow, allowing a formulation of both rate equations and shot noise terms.

The noise terms are zero on average. However, their fluctuations and correlations are given by the transition rates associated with the respective energy reservoir,

$$\begin{aligned}\langle q_N(t)q_N(t + \Delta t) \rangle &= \left( \sigma j + \frac{\beta}{\tau}(n + 1)N + 2\frac{\beta}{\tau}N_Tn + \frac{\beta}{\tau}N \right) \delta(\Delta t) \\ \langle q_n(t)q_n(t + \Delta t) \rangle &= \left( 2\kappa n + 2\frac{\beta}{\tau}N_Tn + \frac{\beta}{\tau}N \right) \delta(\Delta t) \\ \langle q_N(t)q_n(t + \Delta t) \rangle &= - \left( 2\frac{\beta}{\tau}N_Tn + \frac{\beta}{\tau}N \right) \delta(\Delta t).\end{aligned}\tag{2}$$

Pump noise suppression is described by the pump noise factor  $\sigma$ . For the purpose of describing the possibility of squeezing, only the ideal case of  $\sigma = 0$  will be considered. Moreover, it should be noted that the actual output intensity  $I(t)$  of the laser device is a fluctuating quantity given by

$$I(t) = 2\kappa n + q_I(t),\tag{3}$$

where the quantum noise statistics of  $q_I(t)$  read

$$\begin{aligned}\langle q_I(t)q_I(t + \Delta t) \rangle &= 2\kappa n \delta(\Delta t) \\ \langle q_n(t)q_I(t + \Delta t) \rangle &= -2\kappa n \delta(\Delta t) \\ \langle q_N(t)q_I(t + \Delta t) \rangle &= 0.\end{aligned}\tag{4}$$

Within the framework of our general laser model, equations (1) to (4) provide a complete description of the fluctuating laser intensity  $I(t)$  for all time-scales and frequencies. The characteristics of a specific device are determined by only four device parameters, i.e. the spontaneous emission factor  $\beta$ , the excitation lifetime  $\tau$ , the excitation number at transparency  $N_T$  and the cavity loss rate  $\kappa$ . In general, these parameters are given by the gain medium and the cavity design. For semiconductor lasers with an active volume of  $V$ , typical material properties are<sup>17</sup>

$$\begin{aligned}\beta V &\approx 10^{-14}\text{cm}^3 \\ \frac{N_T}{V} &\approx 10^{18}\text{cm}^{-3} \\ \tau &\approx 3 \times 10^{-9}\text{s}.\end{aligned}\tag{5}$$

The cavity loss rate  $\kappa$  should be smaller than the maximal gain in order to achieve lasing. For the parameters above, this corresponds to the condition that

$$\kappa < \frac{\beta N_T}{\tau} \approx 3.33 \times 10^{12} \text{s}^{-1}. \quad (6)$$

Usually, the quality of a laser cavity will not be much higher than needed, so the loss rate for semiconductor laser cavities is typically around  $10^{12} \text{s}^{-1}$ . Consequently, the main device parameter for semiconductor laser diodes is the size as given by  $V$  or, alternatively, by the spontaneous emission factor  $\beta$ . Since the size of typical laser diodes is in the micrometer range, spontaneous emission factors for standard diodes range from  $\beta = 10^{-3}$  for small vertical cavity surface emitting lasers to about  $\beta = 10^{-6}$  for edge emitters. Although larger devices are possible, it becomes increasingly difficult to stabilize a single mode. Consequently, a single mode theory will tend to underestimate the laser noise observed in large semiconductor laser diodes.

The light-current characteristic of a laser device described by equation (1) may be obtained from the time averaged energy flow given by the stationary solution of the rate equations. The stationary carrier number average  $\bar{N}$  and the stationary photon number average  $\bar{n}$  read

$$\begin{aligned} \bar{N} &= \frac{1 + \frac{1}{2n_T}}{1 + \frac{1}{2\bar{n}}} N_T \\ \bar{n} &= \frac{j - j_{th}}{4\kappa} - \frac{1}{4} + \sqrt{\left(\frac{j - j_{th}}{4\kappa} + \frac{1}{4}\right)^2 + \frac{j_{th}}{4\kappa}}, \end{aligned} \quad (7)$$

where the photon number at transparency  $n_T$  and the threshold current  $j_{th}$  are given by

$$\begin{aligned} n_T &= \frac{\beta N_T}{2\kappa\tau} \\ j_{th} &= \lim_{\bar{n} \rightarrow \infty} (1 - \beta) \frac{\bar{N}}{\tau} = 2\kappa \frac{1 - \beta}{\beta} \left( n_T + \frac{1}{2} \right). \end{aligned} \quad (8)$$

The threshold current is defined by extrapolating the asymptotic linear increase of  $\bar{n}(j)$  far above threshold to the threshold region. Since the excitation number  $\bar{N}$  is pinned for  $\bar{n} \rightarrow \infty$ , the extrapolated threshold current is equal to the constant loss rate far above threshold.

In the following, the noise properties of the light field will be discussed with respect to the light field intensity given in terms of the average photon number  $\bar{n}$ . It is therefore useful to define the photon number at threshold by

$$n_{th} = \bar{n}(j_{th}) = \sqrt{\frac{1}{16} + \frac{j_{th}}{4\kappa}} - \frac{1}{4}. \quad (9)$$

Note that the order of magnitude for both threshold current  $j_{th}$  and threshold photon number  $n_{th}$  is defined by the spontaneous emission factor  $\beta$ . For  $\beta \ll 1$  and  $n_T = 3/2$ , the threshold current  $j_{th}$  given by equation (8) is equal to  $4\kappa/\beta$  and the photon number at threshold is  $\beta^{-1/2}$ . In terms of electrical currents,  $\kappa = 10^{12} \text{s}^{-1}$  corresponds to  $1.6 \times 10^{-7} \text{A}$ . Therefore, the approximate electrical threshold current  $I_{th}$  of a typical semiconductor laser diode is related to the spontaneous emission factor  $\beta$  by  $2\beta I_{th} \approx 10^{-6} \text{A}$ . This formula allows a simple quantitative estimate of the spontaneous emission factor from the threshold current. For example, a threshold current of 5 mA indicates a spontaneous emission factor of  $\beta = 10^{-4}$ . The noise properties discussed in the following can thus be related directly to the electrical threshold current observed in the light-current characteristics of laser diodes.

### 3. Photon number fluctuations

The noise characteristics of laser light can be investigated by solving the linearized Langevin equations near the stationary solution. The linearization neglects the bilinear modification of the transition rate given by  $2\beta\tau^{-1}\delta N\delta n$ . This modification may become relevant for large correlated fluctuations in both  $n$  and  $N$ . However, even for large  $\delta n$ , the smallness of  $\delta N$  and the lack of correlation between  $\delta n$  and  $\delta N$  usually justify the linear approximation. The linearized dynamics of the fluctuations  $\delta N = N - \bar{N}$  and  $\delta n = n - \bar{n}$  read

$$\begin{aligned}
\frac{d}{dt}\delta N &= -\Gamma_N\delta N - r\omega_R\delta n + q_N(t) \\
\frac{d}{dt}\delta n &= r^{-1}\omega_R\delta N - \gamma_n\delta n + q_n(t),
\end{aligned} \tag{10}$$

where the relevant time-scales are given by the electronic relaxation rate  $\Gamma_N$ , the optical relaxation rate  $\gamma_n$ , and the coupling frequency  $\omega_R$ . The hole-burning ratio  $r$  scales the interaction between photon fluctuations and excitation fluctuations. The four parameters characterizing the fluctuation dynamics are functions of the device properties and the average photon number  $\bar{n}$ , which read

$$\begin{aligned}
\Gamma_N &= \frac{1}{\tau}(1 + 2\beta\bar{n}) \\
\gamma_n &= 2\kappa \frac{n_T + \frac{1}{2}}{\bar{n} + \frac{1}{2}} \\
\omega_R &= 2\kappa \sqrt{\beta \frac{\bar{n} - n_T}{\kappa\tau}} \\
r &= \sqrt{\frac{\kappa\tau}{\beta} \frac{(\bar{n} - n_T)}{(\bar{n} + \frac{1}{2})^2}}.
\end{aligned} \tag{11}$$

The complete set of two time correlation functions describing the temporal fluctuations may now be derived analytically by obtaining the response function of the linear dynamics and applying it to the statistics of the noise input components  $q_N(t)$  and  $q_n(t)$ . However, it is usually possible to identify the major dynamical processes observable in the fluctuation dynamics by concentrating only on the fastest time-scales. In particular, three regimes may be distinguished:

**I. Relaxation oscillations,  $\omega_R \gg \Gamma_N + \gamma_n$**

If the coupling frequency  $\omega_R$  is much larger than the relaxation rates  $\Gamma_N$  and  $\gamma_n$ , the fluctuation dynamics is described by relaxation oscillations with a frequency of  $\omega_R$  and a relaxation rate of  $(\Gamma_N + \gamma_n)/2$ .

**II. Optical relaxation,  $\gamma_n \gg \Gamma_N + \omega_R$**

If the optical relaxation rate  $\gamma_n$  is much larger than the electronic relaxation rate  $\Gamma_N$  and the coupling frequency  $\omega_R$ , the excitation dynamics has no significant effect on the fluctuation dynamics of the light field. The fluctuation dynamics is then approximately described by thermal fluctuations with a coherence time of  $\gamma_n^{-1}$ .

**III. Adiabatic hole-burning,  $\Gamma_N \gg \gamma_n + \omega_R$**

If the electronic relaxation rate  $\Gamma_N$  is much larger than the optical relaxation rate  $\gamma_n$  and the coupling frequency  $\omega_R$ , the excitation number quickly relaxes to the stationary value defined by the much slower photon number fluctuations. This stationary value of the excitation number acts back on the photon number fluctuation through the coupling rate  $\omega_R$ , increasing the relaxation rate in the light field by  $\omega_R^2/\Gamma_N$ . The fluctuation dynamics is then given by exponentially damped fluctuations which are thermal for  $\gamma_n\Gamma_N > \omega_R^2$  and become sub-thermal for  $\gamma_n\Gamma_N < \omega_R^2$ . In large lasers, this solution is typically valid close to threshold.

Figure 2 shows the operating regimes corresponding to the three cases given above as a function of spontaneous emission factor  $\beta$  and average photon number  $\bar{n}$ .

Since quantum optics textbooks often characterize the light field not by two time correlations but by the stationary photon number distribution in the laser cavity, it is interesting to analyze the magnitude of the fluctuations given by the variance  $\langle \delta n^2 \rangle$ . Using equations (2) and (10), an analytical expression can be derived for the photon number fluctuations. For  $\beta \ll 1$  and  $\bar{n} \gg n_T$ , it reads

$$\frac{\delta \bar{n}^2}{\bar{n}^2} \approx \left( 1 + \frac{2\beta\bar{n}^3 (2\beta\bar{n} + 1)}{(n_T + \frac{1}{2})(4\beta(\kappa\tau + 1)\bar{n}^2 + \bar{n} + 2\kappa\tau(n_T + \frac{1}{2}))} \right)^{-1}. \tag{12}$$

Figure 3 shows a contour plot of the fluctuations as a function of spontaneous emission factor  $\beta$  and photon number  $\bar{n}$  for  $3\kappa\tau = 10^4$  and  $n_T = 3/2$ . It should be noted that the thermal noise region with  $\delta n^2 \approx \bar{n}^2$  extends far beyond

the laser threshold for  $\beta > 10^{-6}$ . If the photon number noise threshold  $n_\delta$  is defined as the point at which the photon number fluctuations drop to one half of the thermal noise level, this threshold is given by

$$\frac{2\beta n_\delta^3 (2\beta n_\delta + 1)}{(n_T + \frac{1}{2})(4\beta(\kappa\tau + 1)n_\delta^2 + n_\delta + 2\kappa\tau(n_T + \frac{1}{2}))} = 1. \quad (13)$$

This definition of the threshold may be approximated by distinguishing three types of laser devices, depending on the magnitude of the spontaneous emission factor  $\beta$ . The three laser types are

Type 1: **Macroscopic lasers**, defined by  $\beta^{-1} > 2(2\kappa\tau)^2(n_T + 1/2)$ , with a noise threshold  $n_\delta = n_{th}$  identical with the laser threshold given by equation (9).

Type 2: **Mesoscopic lasers**, defined by  $4\kappa\tau(n_T + 1/2) < \beta^{-1} < 2(2\kappa\tau)^2(n_T + 1/2)$ , with a noise threshold  $n_\delta = 2\kappa\tau(n_T + 1/2) > n_{th}$  slightly above the laser threshold given by equation (9).

Type 3: **Microscopic lasers**, defined by  $\beta^{-1} < 4\kappa\tau(n_T + 1/2)$ , with a noise threshold  $n_\delta = (\kappa\tau/2)n_{th}$ , significantly higher than the laser threshold given by equation (9).

Mesoscopic and microscopic lasers therefore have thermal photon number statistics even above threshold. Recent experimental studies conducted independently on a solid state laser system seem to confirm this result<sup>18</sup>. Note that almost all semiconductor laser diodes fall into these two categories, since the parameters given by equation (5) indicate that macroscopic semiconductor lasers must have a spontaneous emission factor  $\beta$  smaller than  $10^{-8}$ , which corresponds to a threshold current of no less than 50 A. The borderline between mesoscopic and microscopic semiconductor laser devices is found at around  $\beta = 10^{-4}$  or 5 mA threshold current. Thus, most modern semiconductor laser devices should exhibit thermal photon number fluctuations even above the laser threshold.

Microscopic lasers show thermal fluctuations even above quantum efficiencies greater than 50% ( $2\kappa\bar{n} = j_{th}$ ). Since high quantum efficiency is the key to squeezing the laser output by suppressing the pump noise, such devices can produce a squeezed light output even in the presence of thermal photon number fluctuations in the laser cavity.

#### 4. Low frequency noise

Naturally, it is not possible to measure the light field inside the cavity. Nevertheless most quantum theories tend to concentrate on the state of the cavity field modes without any realistic assumptions on the emission process. It is one of the merits of the original work on squeezing the laser output<sup>6</sup> that it points out the practical relevance of distinguishing between the light inside the cavity and the light emitted by the laser device. On short time-scales, this difference may seem to be irrelevant, because the emission process only adds some partition noise for times shorter than  $\kappa^{-1}$  and otherwise produces the same type of statistics as the field inside the cavity. However, energy conservation introduces a constraint at longer time-scales. Because of this external constraint, the fluctuations of the photon current outside the cavity may actually be lower than the photon number fluctuations inside<sup>19</sup>. If the excitation dynamics are eliminated, this constraint affects both the photon number fluctuations inside the cavity and the low frequency noise equally<sup>14</sup>. If the excitations in the gain medium provide an additional energy reservoir, however, care must be taken to distinguish the long term effects of such energy storage on the low frequency noise from the short term effects of energy exchange between the gain medium and the light field on the photon number fluctuations.

The field outside the cavity represents energy lost from the laser device, while a time average over the field inside the cavity does not have this meaning. In particular, a single photon might stay inside the cavity for a long time or for a short time - in the field outside, it will only appear once. In order to describe the low frequency part of intensity fluctuations, it is therefore necessary to discuss the output intensity  $I(t)$  introduced in equation (3).

The fluctuations of the average intensity  $\bar{I} = 2\kappa\bar{n}$  are given by

$$\delta I(t) = I(t) - \bar{I} = 2\kappa\delta n + q_I(t). \quad (14)$$

The two time correlation function of the intensity fluctuation is then given by

$$\langle \delta I(t) \delta I(t + \Delta t) \rangle = 4\kappa^2 \langle \delta n(t) \delta n(t + \Delta t) \rangle + 2\kappa \langle q_I(t) \delta n(t + \Delta t) \rangle + \langle q_I(t) q_I(t + \Delta t) \rangle. \quad (15)$$

The last term represents the shot noise level  $L_{\text{SN}}$  induced by quantum fluctuations at the cavity mirrors. It is therefore convenient to use this term as a normalization term when performing the time average representing the limit of low frequencies,

$$\frac{\overline{\delta I^2}(\omega \rightarrow 0)}{L_{\text{SN}}} = \frac{\int_0^\infty d\tau (4\kappa^2 \langle \delta n(t) \delta n(t+\tau) \rangle + 2\kappa \langle q_I(t) \delta n(t+\tau) \rangle)}{\int_0^\infty d\tau \langle q_I(t) q_I(t+\tau) \rangle} + 1. \quad (16)$$

Using this normalization to the shot noise level, the low frequency limit of the intensity noise can be calculated using the linearized Langevin equation (10) with the noise terms given by equations (2) and (4). For  $\beta \ll 1$  and  $\bar{n} \gg n_T$ , the result reads

$$\frac{\overline{\delta I^2}(\omega \rightarrow 0)}{L_{\text{SN}}} \approx \frac{(n_T + \frac{1}{2}) (4\beta \bar{n}^3 + 2\bar{n}^2 + (n_T + \frac{1}{2}))}{(2\beta \bar{n}^2 + (n_T + \frac{1}{2}))^2} + \sigma \frac{4\beta^2 \bar{n}^4 + 4\beta(n_T + \frac{1}{2})\bar{n}^3}{(2\beta \bar{n}^2 + (n_T + \frac{1}{2}))^2}. \quad (17)$$

Figure 4 illustrates the low frequency noise characteristics for various levels of pump noise suppression  $\sigma$ . Note that the low frequency noise below about two times threshold current ( $2\kappa\bar{n} = j_{th}$ ) does not depend very much on pump noise suppression. Moreover, the peak value of low frequency noise always coincides with the laser threshold. Above two times threshold, however, the squeezing obtained for  $\bar{n} \rightarrow \infty$  is given by  $\sigma$ . In the following, the maximal squeezing potential represented by the  $\sigma = 0$  result will be investigated.

Since squeezing is defined by intensity noise below the shot noise limit, the squeezing threshold  $n_{sq}$  for  $\sigma = 0$  can be defined as the point at which  $\overline{\delta I^2}(\omega \rightarrow 0) = L_{\text{SN}}$ . Using  $\beta \ll 1$  and  $n_T > 1/2$ , this threshold is found to be given by

$$n_{sq} = \frac{1}{2\beta} \left( (n_T + \frac{1}{2}) + \sqrt{(n_T + \frac{1}{2})^2 + (n_T + \frac{1}{2})} \right) \approx \frac{n_T + \frac{1}{2}}{\beta} \approx \frac{j_{th}}{2\kappa}. \quad (18)$$

The squeezing threshold is thus found at two times threshold current, corresponding to a quantum efficiency of 50%.

## 5. Coexistence of squeezing and thermal noise

Above two times threshold ( $2\kappa\bar{n} = j_{th}$ ), the low frequency noise component of a semiconductor laser device may be squeezed below the shot noise level by suppressing the noise in the injected current. All the same, microscopic devices with spontaneous emission factors  $\beta$  above  $10^{-4}$  and corresponding threshold currents below 5 mA still exhibit thermal photon number fluctuations on short time-scales. Therefore, an operating regime exists in which squeezing and thermal noise coexist in the same light field emission. This regime is defined as the region between the squeezing threshold given in equation (18) and the noise threshold given in equation (13). The laser threshold and the two fluctuation thresholds are shown in figure 5. At about  $\beta = 10^{-3}$ , the two thresholds cross. Therefore, coexistence of thermal noise and squeezing can be observed in devices with  $\beta > 10^{-3}$ . Figure 6 shows the photon number fluctuations and the low frequency noise for a mesoscopic ( $\beta = 10^{-4}$ ) and a microscopic ( $\beta = 10^{-2}$ ) laser device. In the microscopic device, coexistence of thermal noise and squeezing is clearly observable just above two times threshold ( $2\kappa\bar{n} = j_{th}$ ).

What is the relationship between the thermal photon number fluctuations inside the cavity observed on picosecond time-scales and the suppression of noise below the shot noise limit on time-scales longer than the relaxation rates of the laser dynamics? An attempt to visualize this relationship is shown in figure 7. The short term fluctuations average out as the temporal average is taken. This effect is due to the anti-correlation of fluctuations at intermediate time differences. In the case of over-damped relaxation oscillations, the short term fluctuations are given by a fast optical relaxation  $\gamma_n$  and a much slower relaxation of the carrier system  $\Gamma_N$ . At a quantum efficiency of 50 % ( $2\kappa\bar{n} = j_{th}$ ), the two time correlations of  $I(t)$  is approximately given by

$$\langle \delta I(t) \delta I(t + \Delta t) \rangle \approx 2\kappa\bar{n}\delta(\Delta t) + 4\kappa^2\bar{n}^2 \exp(-\gamma_n\Delta t) - 4\kappa^2\bar{n}^2 \frac{\Gamma_N}{\gamma_n} \exp(-\Gamma_N\Delta t). \quad (19)$$

While the time integrated contribution of the short time bunching is exactly equal to the time integrated contribution of the long time anti-bunching, the thermal bunching contributions dominate near  $\Delta t = 0$  by a ratio equal to the

time-scale ratio  $\gamma_n/\Gamma_N \gg 1$ . The total low frequency noise is then equal to the shot noise term only, because the thermal fluctuations are anti-correlated on a time-scale of  $1/\Gamma_n \approx \tau$  by the slow relaxation dynamics of the excitations in the gain medium. Figure 8 illustrates this transition from bunching to anti-bunching in the two time correlation of the laser output.

Note that the optical coherence time is equal to or even smaller than  $1/\gamma_n$ . In particular, the line-width enhancement effects described by the  $\alpha$  factor and the Peterman factor, respectively, are known to cause an additional reduction of the phase coherence not related to the photon number relaxation. Therefore, the first order coherence time will be much shorter than the time during which thermal bunching is observed in the photon number statistics. This lack of first order coherence suggests that the squeezing of low frequency intensity noise below the shot noise limit is only obtained by a summation of the light field intensities of many modes with independent phase fluctuations. This type of collective squeezing cannot be attributed to a single coherent light field mode. Instead, it should be considered a multi-mode property.

## 6. Implications for quantum optics

The light emitted by a laser propagates in the multi-mode continuum of the unconfined electromagnetic field. It is therefore difficult to describe it in terms of a discrete mode structure. As a result, photon number measurements cannot be assigned to well defined modes. The randomness of photon detection events is a consequence of this conceptual difficulty. Nevertheless, time integrated measurements of photon number can provide precise information about the number of photons within a given volume. It is tempting to associate this collective information directly with the single mode photon number. However, the lack of information available about the actual photon number distribution among the many modes within the observed volume should be considered as well. In particular, if all possible photon number distributions of  $n$  photons among  $M$  modes are considered to be equally likely, the situation corresponds to the micro-canonical ensemble of thermodynamics<sup>20</sup>. The density matrix of any mode  $i$  which is an arbitrary superposition of the  $M$  modes then reads

$$\hat{\rho} = \frac{N!(M-1)}{(N+M-1)!} \sum_{n=0}^N \frac{(M+N-2-n)!}{(N-n)!} |n\rangle\langle n| \approx \frac{M}{N+M} \sum_{n=0}^{\infty} \left(1 + \frac{M}{N}\right)^{-n} |n\rangle\langle n|, \quad (20)$$

where the approximation is for large  $M$  and  $N$ . The photon number distribution of every single mode corresponds to a thermal distribution at a temperature proportional to the inverse logarithm of  $1 + M/N$  even though the total photon number in the  $M$  modes is given precisely by  $N$ . While the photon number in each mode fluctuates thermally, the fluctuations in the total photon number are suppressed by anti-correlations between the modes. Such anti-correlations have been described theoretically<sup>13,16</sup> and observed experimentally between different longitudinal modes<sup>10,15</sup> and between orthogonal polarizations<sup>12</sup> in the squeezed light emission from semiconductor lasers. As the discussion in this paper indicates, it should be observable in the temporal mode structure as well. For the case of over-damped relaxation oscillations, equation (19) shows the anti-correlations between emission modes separated by a time roughly equal to the excitation lifetime  $1/\Gamma_N$ . If the coherence length is assumed to be about  $1/\gamma_n$ , the intensity distribution given by equation (19) might be interpreted according to equation (20) with  $M = \gamma_n/\Gamma_N$  and  $N = 2\kappa\bar{n}/\Gamma_N$ . Thus the light field statistics given by equation (19) suggest that not a single mode of the total light field emission is in a squeezed state. If the low frequency noise is squeezed below the shot noise limit, this effect can be explained as a purely collective property without implications for any particular mode. Specifically, the slight anti-correlation in photon number between different modes shown by the two time correlation function given in equation (19) above does not represent quantum mechanical entanglement, since there is virtually no phase correlation between the respective modes. A semi-classical interpretation of the intensity noise is therefore sufficient to explain the squeezing properties of the laser field.

## 7. Conclusions

We have shown that thermal noise in laser cavities may coexist with squeezing in the low frequency intensity fluctuations. The reason for this coexistence is that squeezing is caused by the long term anti-correlation of fluctuations

caused by the slow loss rate of excitations from the gain medium, while the light field fluctuations within the optical coherence time are dominated by stimulated emission and photon bunching. Consequently, squeezing the low frequency intensity fluctuations of a semiconductor laser diode does not usually reduce the photon number fluctuations of any single mode. Instead, only the total photon number of a large number of modes is controlled. This type of collective squeezing should be distinguished from the single mode squeezing obtained e.g. by optical parametric amplification. While squeezing in semiconductor laser diodes represents a significant achievement in controlling the energy flow of light on the quantum level, it does not produce the entanglement properties which are typically observed in the single mode squeezing of optical parametric amplifiers. This limitation severely restricts the use of collectively squeezed light in quantum optical applications.

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Fig. 1. Energy flow diagram of a single mode laser

Fig. 2. Dominant timescales of the fluctuation dynamics by average photon number  $\bar{n}$  and spontaneous emission factor  $\beta$  for  $n_T = 3/2$  and  $3\kappa\tau = 10^4$ . The threshold photon number  $n_{th}$  is given by the dotted line.

Fig. 3. Contour plot of the photon number fluctuations as a function of average photon number  $\bar{n}$  and spontaneous emission factor  $\beta$  for  $n_T = 3/2$  and  $3\kappa\tau = 10^4$ . The contours correspond to constant ratios of the photon number fluctuations  $\delta\bar{n}^2$  and the shot noise level  $\bar{n}$ . This ratio is equal to one in the black region and increases by a factor of  $10^{5/9} \approx 3.6$  for every contour. The initial increase at low photon numbers  $\bar{n}$  is thermal ( $\delta\bar{n}^2 \approx \bar{n}^2$ ).

Fig. 4. Low frequency noise characteristics in dB relative to the shot noise limit for  $n_T = 3/2$  and  $\beta = 10^{-3}$  and pump noise factors of  $\sigma = 1$ ,  $\sigma = 0.25$ ,  $\sigma = 0.0625$  and  $\sigma = 0$ .

Fig. 5. Noise threshold  $n_\delta$  and squeezing threshold  $n_{sq}$  as a function of the device size given by the inverse spontaneous emission factor  $\beta^{-1}$ . The other device parameters are constant at  $n_T = 3/2$  and  $3\kappa\tau = 10^4$ .

Fig. 6. Photon number fluctuations (PNF) and low frequency intensity noise (LFN) characteristics in dB relative to shot noise for  $n_T = 3/2$  and  $3\kappa\tau = 10^4$ . (a) shows a mesoscopic laser with  $\beta = 10^{-4}$  and (b) shows a microscopic laser with  $\beta = 10^{-2}$ . For comparison with the injected current, the dashed lines mark the region between twice threshold ( $2\kappa\bar{n} = j_{th}$ ) and ten times threshold ( $2\kappa\bar{n} = 9j_{th}$ ).

Fig. 7. Visualization of the intensity noise. The noise averages out on long time-scales, even though it is thermal on short time-scales.

Fig. 8. Two time correlation of intensity  $\langle\delta I(t)\delta I(t + \Delta t)\rangle$  for  $\gamma_n/\Gamma_n = 5$ .

Figure 1

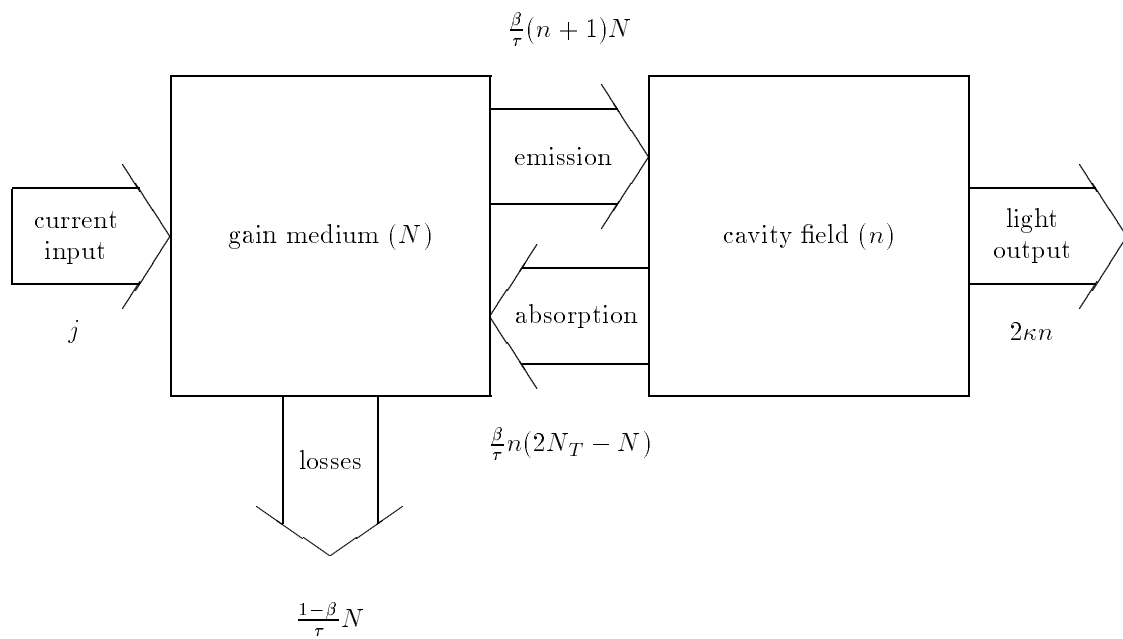


Figure 2

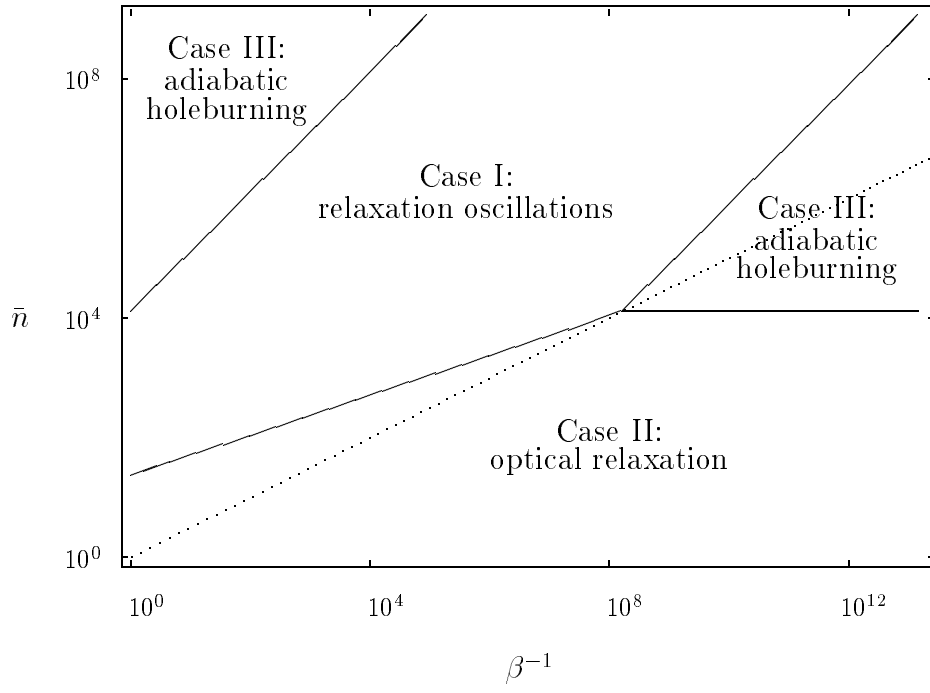


Figure 3

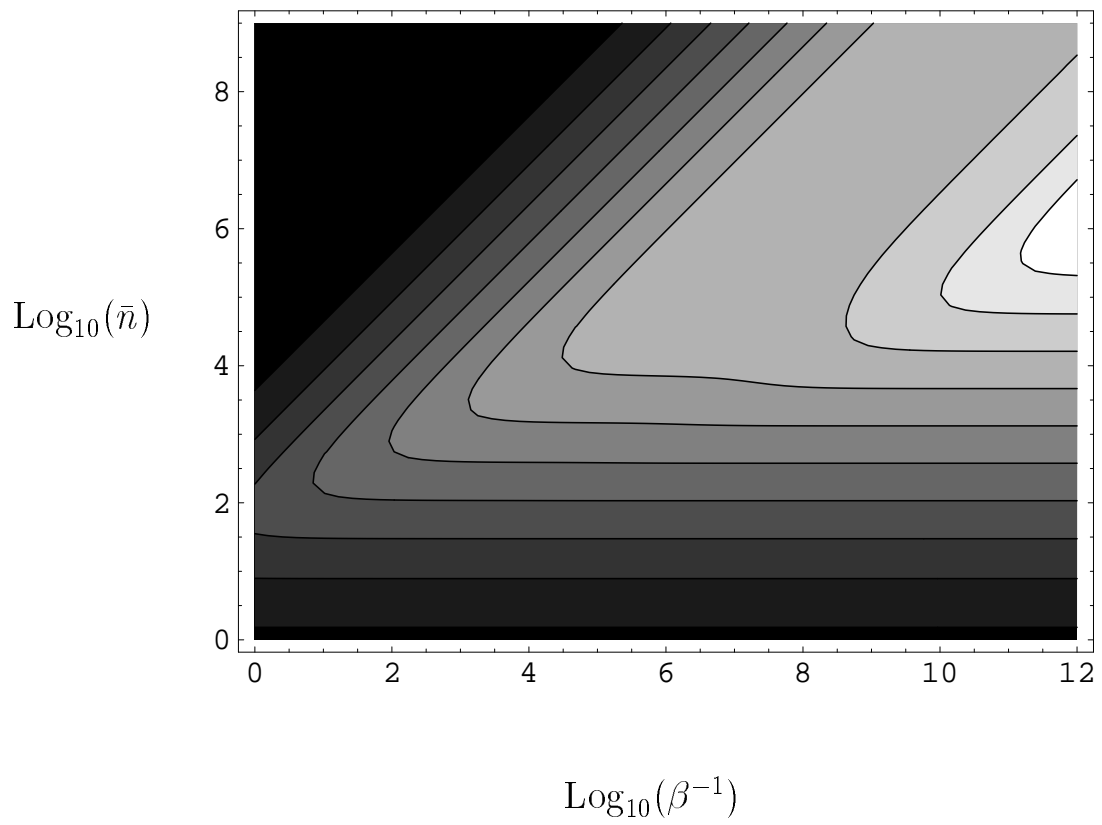


Figure 4

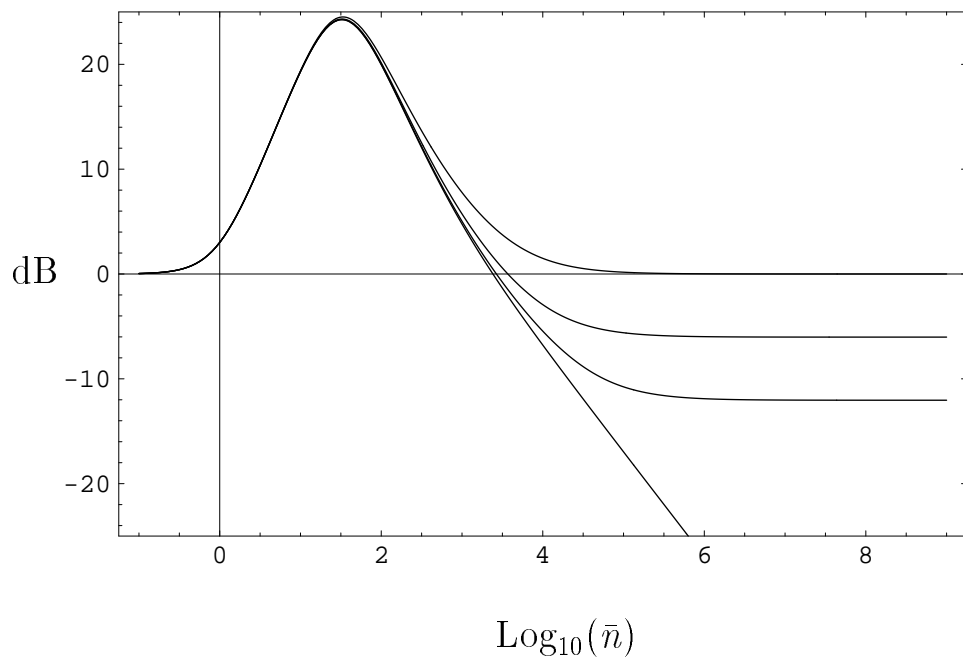


Figure 5

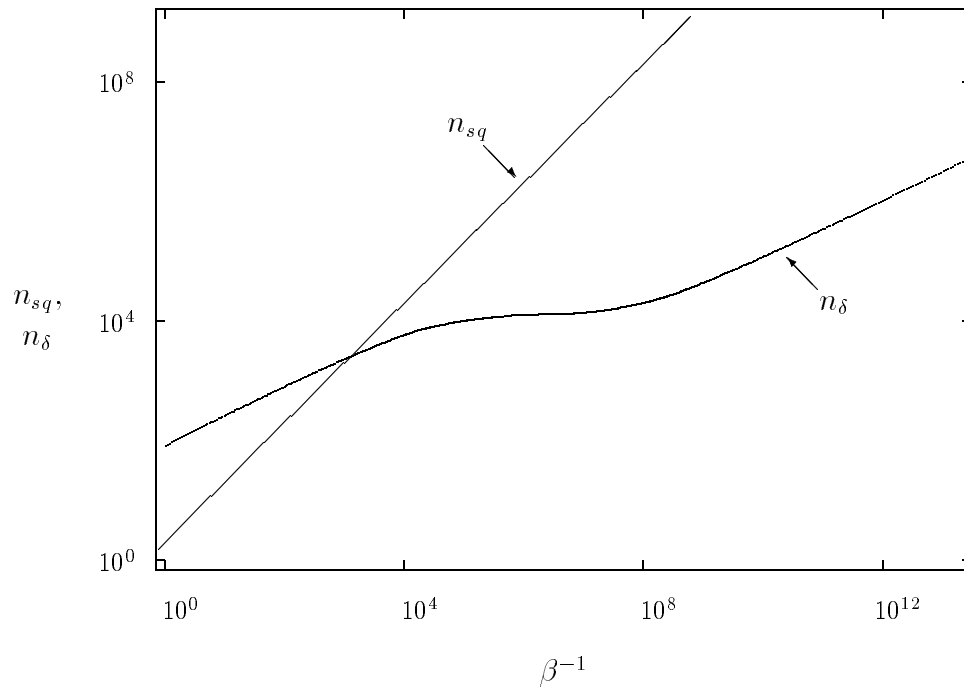


Figure 6

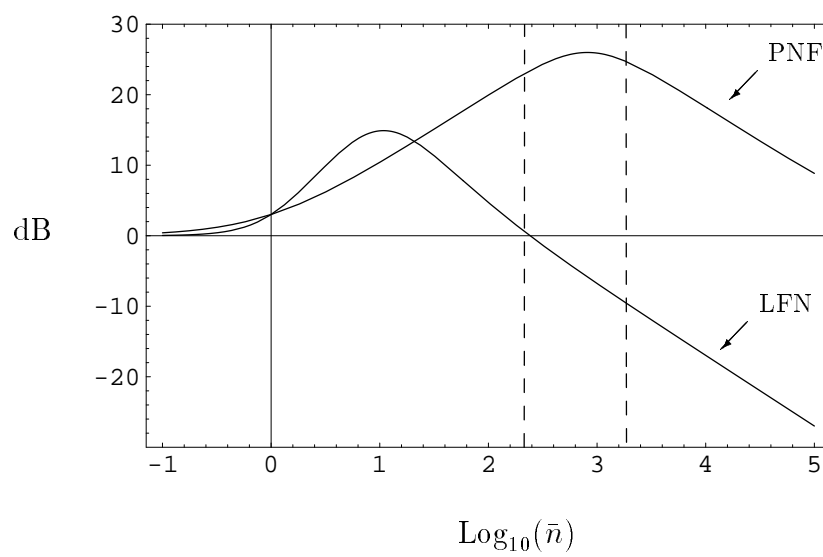
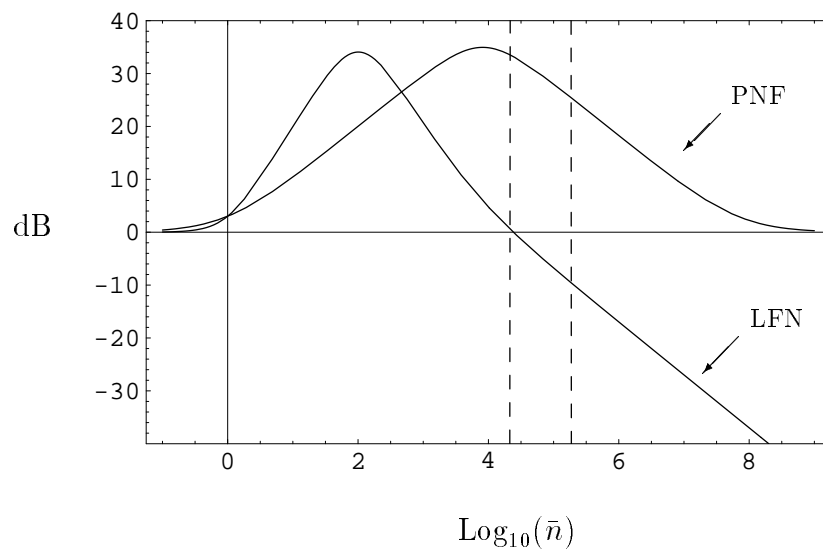


Figure 7

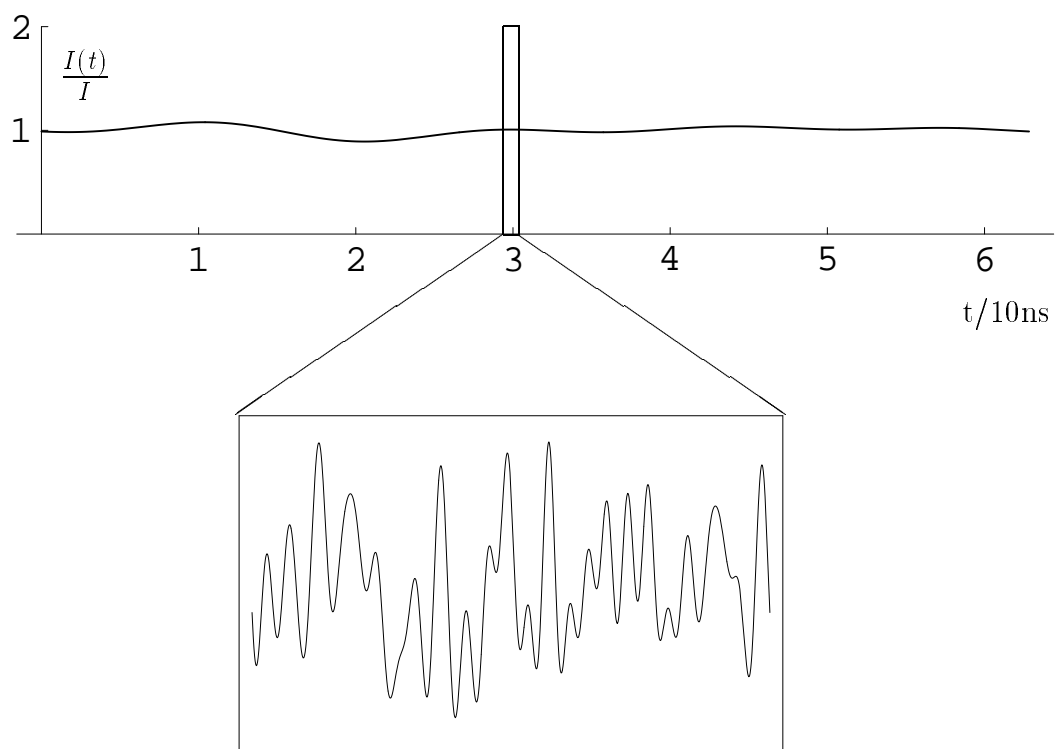




Figure 8

